

Tips for Solving Math Problems in APES (AP Strategies)

Energy Efficiency

$$\text{Efficiency} = (\text{Work Output}/\text{Work Input}) \times 100\%$$

Efficiency refers to the ability of an energy conversion reaction to convert energy from one form to the next without losing too much heat to the environment (electrical to light, chemical to mechanical, and etc.)

An energy conversion that doesn't lose any heat at all would have a 100% efficiency rating.

If you have to determine the work input of a reaction and you are given the energy efficiency and work output, then divide the output by the efficiency percentage.

Dimensional Analysis Review

1) What units of measure do you want in the answer?

In this problem you want to know how many BTU's are needed each day. After you figure out what units you want to know, translate the English into Math. In Math terms, what you want to know is:

$$\frac{\text{BTU's}}{\text{Day}}$$

2) What do you know?

What do you know about "BTU's" or "days"? You know that there are 12, 000, 000 kWh produced in a day (this will be provided in a problem). You also know that if 1kWh is produced by 10, 000 BTU's. Connecting kWh from the two equivalencies, bridges the information we know to the information requested in the problem. When you have this kind of connection between units, then you know enough to solve the problem – but first translate what you know into math terms that you can use when solving the problem.

$$\frac{12,000,000 \text{ kWh}}{\text{Day}} \quad \text{or} \quad \frac{\text{day}}{12,000,000 \text{ kWh}} \quad \text{or} \quad \frac{1 \text{ kWh}}{10,000 \text{ BTU}} \quad \text{or} \quad \frac{10,000 \text{ BTU}}{1 \text{ kWh}}$$

3) Pick from these statements the ones that you actually need.

The trick is to pick the equivalencies that will cancel out the unit you don't want. You want "BTU's" on top in your answer. On the bottom you want "days". You need to get rid of the "kWh". You cancel "kWh" out by picking a factor that has "kWh" on top and one that has "kWh" on the bottom. This allows this unit to be cancelled out.

$$\frac{12,000,000 \text{ kWh}}{\text{Day}} \quad \times \quad \frac{10,000 \text{ BTU}}{1 \text{ kWh}}$$

4. Solve it.

When you have cancelled out the units you don't want and are left only with the units you do want, then you know it's time to multiply all the top numbers together, and divide by all the bottom numbers.

$$\frac{12,000,000 \text{ kWh}}{\text{Day}} \quad \times \quad \frac{10,000 \text{ BTU}}{1 \text{ kWh}}$$

Scientific Notation

1. Converting to Scientific Notation:

$$0.00234 = 2.34 \times 10^{-3}$$

Write the number .00234 as a coefficient. Coefficients need to be between 1 and 10. The coefficient is then multiplied by ten raised by an exponent. The number of places to the left that you move the decimal point is the exponent. If you move the decimal to the right, the exponent will be negative.

$$\begin{aligned} &= 0.0234/10 \\ &= 0.234 / (10 \times 10) \\ &= 2.34 / (10 \times 10 \times 10) \\ &= 2.34 / (10^3) \\ &= 2.34 \times 10^{-3} \end{aligned}$$

2. Adding and subtracting two numbers written in scientific notation:

$$5.2 \times 10^3 + 3.6 \times 10^4 = 4.12 \times 10^4$$

Factor out one of the powers of ten; usually the smaller one is the easiest. Divide both numbers by the power of ten and multiplying the whole quantity by the same power of ten. To divide one power of ten by another, simply subtract the two exponents. Next, convert the two numbers from scientific notation to real numbers. Now add the two numbers normally. Finally convert to scientific notation if the coefficient is less than 1 or greater than 10.

$$\begin{aligned} &5.2 \times 10^3 + 3.6 \times 10^4 \\ &= (5.2 \times 10^3/10^3 + 3.6 \times 10^4/10^3) \times 10^3 \\ &= (5.2 \times 10^0 + 3.6 \times 10^1) \times 10^3 \\ &= (5.2 + 36) \times 10^3 \\ &= 41.2 \times 10^3 \\ &= 4.12 \times 10^4 \end{aligned}$$

$$3.9 \times 10^{-6} - 6.9 \times 10^{-5} = -6.51 \times 10^{-5}$$

Factor out one of the powers of ten. Next, convert both scientific notation numbers to real numbers. Subtract the two numbers normally and convert to scientific notation if the coefficient is not between 1 and 10 (or -1 and -10).

$$\begin{aligned} &3.9 \times 10^{-6} - 6.9 \times 10^{-5} \\ &= (3.9 \times 10^{-6}/10^{-5} - 6.9 \times 10^{-5}/10^{-5}) \times 10^{-5} \\ &= (3.9 \times 10^{-1} - 6.9 \times 10^0) \times 10^{-5} \\ &= (.39 - 6.9) \times 10^{-5} \\ &= -6.51 \times 10^{-5} \end{aligned}$$

3. Multiplying two numbers written in scientific notation:

$$(4 \times 10^{-2}) \times (2 \times 10^{10}) = 8 \times 10^8$$

Multiply the two coefficients and then multiply the two powers of ten by adding their exponents: since $-2 + 10 = 8$, then $10^{-2} \times 10^8$. Finally, combine your two answers and convert to scientific notation: 8×10^8 .

$$\begin{aligned} &(4 \times 10^{-2}) \times (2 \times 10^{10}) \\ &= (4 \times 2) \times (10^{-2} \times 10^{10}[\text{add}]) \\ &= (8) \times (10^8) \\ &= 8 \times 10^8 \end{aligned}$$

4. Divide two numbers written in scientific notation:

$$(4.2 \times 10^{-6}) / (6 \times 10^{-2}) = 7 \times 10^{-5}$$

Divide the two coefficients: $4.2/6 = 0.7$. Then, divide the two powers of ten by subtracting their exponents: Since $-6 - (-2) = -6 + 2 = -4$, then $10^{-6}/10^{-2} = 10^{-4}$. Finally, combine your two answers and convert to scientific notation.

$$(4.2 \times 10^{-6}) / (6 \times 10^{-2})$$

$$= (4.2/6) \times 10^{-6}/10^{-2} \text{ [subtract]}$$

$$= (0.7) \times (10^{-4})$$

$$= 7 \times 10^{-5}$$

Population Growth Rate

Growth rate is calculated by the change in the population divided by the total population, then multiplied by 100 to report the change in a percentage.

When given the CBR and the CDR use the following equation:

$$\frac{\text{CBR} - \text{CDR}}{10} = \%$$

Rule of 70

Calculates the doubling time based upon a percentage.

$$\frac{70}{\% \text{ growth}} = \text{years to double}$$

% growth (do not convert to a decimal)

Per capita

Data given divided by total population